SOROBAN ABACUS HANDBOOK

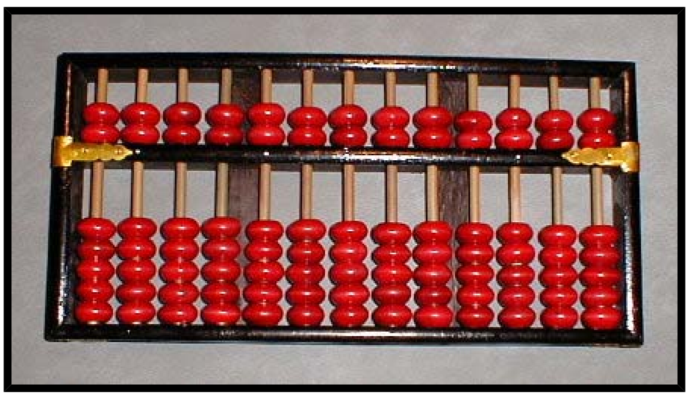
The Soroban / Abacus Handbook
is © 2001-2005 by David Bernazzani
Rev 1.05 - March 2, 2005

INTRODUCTION

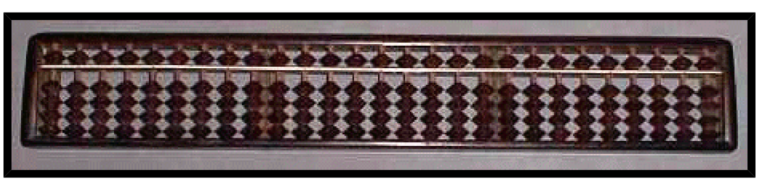
The Abacus (or Soroban as it is called in Japan) is an ancient mathematical instrument used for calculation. The Abacus is one of the worlds first real calculating tools – and early forms of an Abacus are nearly 2500 years old. The word Abacus is derived from the Greek "Abax" meaning counting board and the original types of Abacus were stone slates with dust covering them and a stylus used for marking numbers. Later this evolved into a slate with groves where rocks or other counters would be placed to mark numbers. Later it finally evolved into a framed device with beads sliding along bamboo rods.

I have always been fascinated by the abacus – and have recently taken up the study of this tool. I have practiced on both a Chinese Abacus (called a *Suan Pan*) and on a Japanese *Soroban*. The modern Chinese abacus has been in use since about the 14th century. The Japanese Soroban has been in use since at least the 16th century. I much prefer the Japanese Soroban as a more aesthetically pleasing and a more efficient calculating instrument. There are some key differences between the two types of instruments.

Here is a picture of a traditional Chinese Abacus. As you can see this instrument has 2 beads above the reckoning bar and 5 beads below it.



And here is a picture of a traditional Japanese Soroban. Here we have a streamlined instrument with 1 bead above the reckoning bar and 4 below it.



Originally the Japanese Soroban looked much like the Chinese Abacus (5 beads below, 2 beads above) but it was simplified around 1850 and reduced to a single bead above the reckoning bar and later in 1930 to just 4 beads below it.

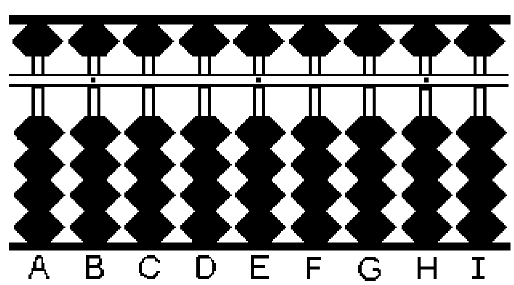
It doesn't matter which type you use – both have the same procedure for recording numbers and performing addition, subtraction, multiplication and division. The Chinese abacus is capable of counting 16 different numbers from 0 to 15 on each individual rod which was useful since their units of weight were (are?) measured in 16ths. For westerners – this is not very useful unless you want to do calculations in 16ths of an inch or maybe hexadecimal (for computer programs which is base 16). Of course you don't have to use all the beads –

so you can always just represent numbers 0-9 which is most useful for our purposes. The Japanese Soroban has been streamlined for the Hindu-Arabic number system and each rod can represent one of 10 different numbers (0-9) and has no wasted beads for our decimal calculations. Read on to discover how numbers are represented on the abacus frame.

I have researched both types and studied different methods for performing operations on the board. I will try to instruct and teach only the best methods that I have found. Generally, these are the methods described by Takashi Kojima in his excellent book "The Japanese Abacus – It's Use and Theory" published first in the 1950's and later reprinted. I assume the reader wants to know the best prescribed procedures for learning how to use the abacus. I will try to take no shortcuts – and hope that this handbook serves as a fine introduction to the marvelous world of the Abacus. One might ask why anyone should bother learning how to use the Abacus with the advent of such cheap calculators. The answers will be different for everyone – but for me it was a desire to learn and understand this ancient skill and to become skilled myself with this fascinating tool. I also find the practice on the Soroban very relaxing – often helping me unwind after a day at work. I suppose it is as much a hobby as anything else. With practice and the help of this handbook, you can master the Abacus! If you don't have an instrument, you can find proven suggestions at the end of this handbook for tracking one down or making one yourself!

USING THE ABACUS

The **beads** on the abacus slide up and down on what we call **rods** which are divided horizontally by a **reckoning bar** (some call this the **beam**). In general, the single row of beads above the reckoning bar (called "Heaven Beads") are worth 5. The beads below the reckoning bar (called "Earth Beads") are worth 1. A bead is said to "have value" when it is pushed towards the middle reckoning bar and "loses value" when moved away from the middle reckoning bar. Forming a number on the abacus is very simple – simply move a bead towards the reckoning bar for it to "have value". Let's look at this virtual abacus frame and "read" the number.



Since no beads are touching the middle reckoning bar, we read a number of 0 all the way across the abacus. This is a "cleared" frame and it is how you normally start every math problem (similar to a reset on a modern calculator). To clear a Soroban – you simply tilt it towards you so that all the beads are pulled by gravity towards the bottom of the frame (this clears the lower beads only!) and then level the instrument again and make a sweeping motion of your index finger between the reckoning bar and the upper row of beads – this will cause them to "push" upwards away from the reckoning bar. It's almost as if you were splitting the beads from the reckoning bar with a parting sweep of your finger. You could just move them up individually with a flick of your index finger – but the sweeping method from left to right is very fast and efficient once you get the hang of it.

Notice the little dots on the reckoning bar. We use these dots on every third column to designate a "unit" rod. You should place your number so that the unit portion falls on this rod (for example ,in the number 1234 the value 4 is the "units", 3 is the "tens", 2 is the "hundreds", etc). Use rods to the right of this unit rod as tenths, hundredths, and rods to the left as tens, hundreds, thousands (which also has a unit dot – very convenient!). It doesn't matter which dot you choose to be the unit rod – so long as it is marked with a dot for easy reference. Numbers can be formed on the abacus anywhere you want – and with some problems (like multiplication and division) there will be two or three sets of numbers on the abacus frame. Do not think of the dot as a decimal point – that might get confusing as to whether that rod is representing the value just before the decimal point or just after it. It is a unit rod – and rods to the right have a factor of 10 less value and rods to the left have a factor of 10 more. For example, lets assume in the diagram above we chose E as our unit rod. We would then have:

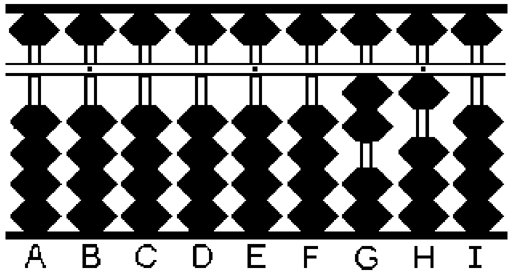
Rod	What it Represents
Rod A	Ten Thousands (10000's)
Rod B	Thousands (1000's)
Rod C	Hundreds (100's)
Rod D	Tens (10's)
Rod E	Units (1's)
Rod F	Tenths (0.1's)
Rod G	Hundredths (0.01's)
Rod H	Thousandths (0.001's)
Rod I	Ten Thousandths (0.0001's)

Before we move on, you have just enough information to understand the way numbers are formed on the frame. I will take the time here to give you some tips for Soroban mastery – some of these won't make sense until later in the handbook – but you can start to apply them as you learn the full techniques for the different operations we will perform on the Abacus.

- Always work numbers on the Abacus from left to right. It's the most efficient way. Don't fall into old habits of trying to add and subtract starting at the right (as you would with pencil and paper).
- When using the Abacus, place it on a level surface – steadying it with your left hand if necessary and work with your right hand (otherwise you partially block and cover numbers needlessly as you work with it).
- Use two fingers for bead manipulation only – the thumb is always used when moving one or more earth beads up to the reckoning bar. The index finger is used for everything else (moving earth beads back away from the reckoning bar and moving heaven beads to and from the reckoning bar). Use just enough force to move the beads – do not slam them which can disrupt beads on neighboring rods.
- When performing addition, always finish moving beads on the current rod before dealing with any carry to the tens rod (which is always the rod to the left of the one you are working on – it will always have 10x the value of the current rod).
- When performing subtraction, always borrow from the tens rod before finishing moving beads on the current rod (which is always the rod to the left of the one you are working on – it will always have 10x the value of the current rod).

ADDITION

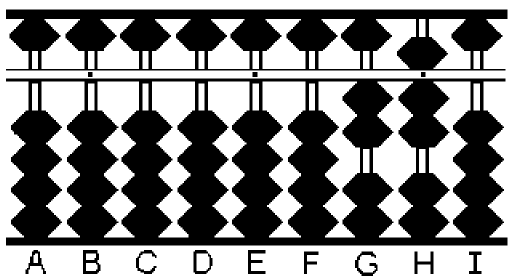
Let's put a real number on the Abacus. Let's put the number **21** on the frame. We conveniently choose rod H as our unit rod and form the number.



Here we have placed the value of 2 on rod G (by moving 2 earth beads worth 1 each towards the reckoning bar) and the value of 1 on rod H (by moving a single earth bead worth 1 each towards the reckoning bar). I will henceforth use shorthand for this and simply say "**Place 21**" which means to place 21 on the frame (remember to observe the unit rod when you place the numbers – it will help you to remember the size and position of the number more easily as the calculations get more complicated).

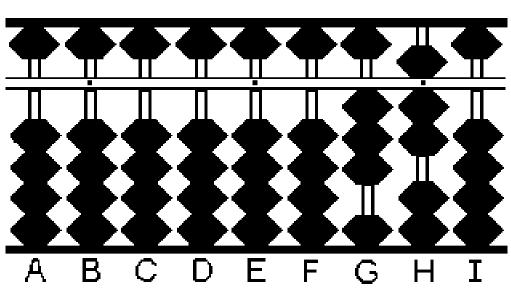
It's important to know that when you enter this number on the frame, you should enter it from left to right. Numbers are read and spoken from left to right and so it's much more efficient to enter them on the frame in the same manner. Again, be sure your number ends up on a unit rod for convenience.

Now let's add **6** to this number.



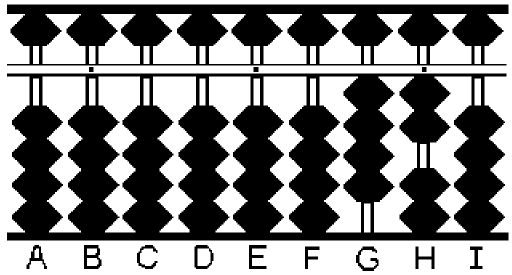
As you can see, we added 6 to the units place of the original number (21) on the frame. We do this by moving 6 worth of beads towards the reckoning bar for rod H. We don't have enough earth beads (worth 1) to do it so we must take a single heaven bead (worth 5) and a single earth bead (worth 1) to make the 6 we need. Using the proper fingering techniques described before – you use your thumb to move up the singe earth bead and your index finger to move down the single heaven bead. This can be done in one motion – almost a "squeezing" effect that is very efficient. After moving those beads towards the reckoning bar (henceforth in this document called "**Add 6**") you can now "read" the value resulting on the frame. Rod G has a value of 2 and rod H (our unit rod) has a value of 7. The answer, therefore, is 27. There, you've just added your first numbers on the Soroban!

Now, let's get tricky! We will take this resulting answer (27) and add **15** to it. Starting with the 1 in "15" we want to put this on Rod G – so we simply "Add 1" by moving a single bead towards the reckoning bar. See the following resulting frame which now contains the value of 37 (not the final answer, just the intermediate value after adding the 1 in "15").



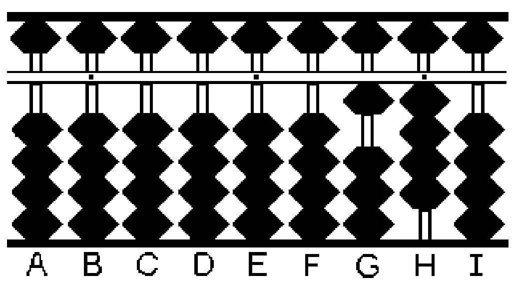
And now we must add the 5 of the "15" (remember, we are working from left to right with these numbers) and so we must "Add 5" on the unit rod (H). Wait! We don't have 5 beads to work with – so instead we "Subtract 5" and "Add 10" which involves a shift to the next column up (to the left which is 10 times the value of the current rod). This is a key concept on the Abacus – we didn't have enough to "Add 5" to our unit column so we must instead "Subtract 5" on the unit rod and then "Add 10" (by moving a single earth bead worth 1 in the next column up – in this case rod G). In this case, always work with the current rod first – moving a single heaven bead away from the reckoning bar on H and then moving up a single earth bead on G.

This results in the following frame:



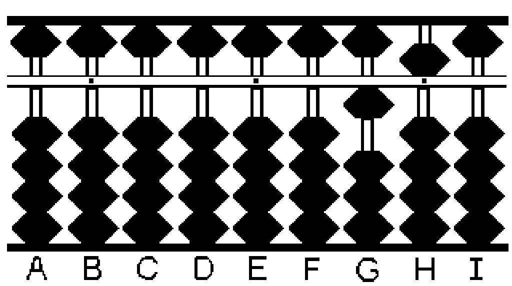
As you can see, the resulting number on GH is **42** which is the result of adding 15 to 27. It is imperative to remember that when wanted to add the 5 to the unit rod H, you did not mentally say "Well… there is 7 already on the rod, and I'm adding 5 so that is 12 – I must somehow form 12 on the frame!". That line of thinking will make abacus processing very slow – the right way to do it is to know that you must "Add 5" on H – if you lack the beads, you must "Add 1" on the next highest rod and "Subtract 5" on the unit rod which accomplishes the same thing. This sort of mechanical processing is quick to learn and allows for incredible efficiency when working with the Soroban.

The same borrowing processing results even within a single rod. Let's look at the following number **14** on the frame:



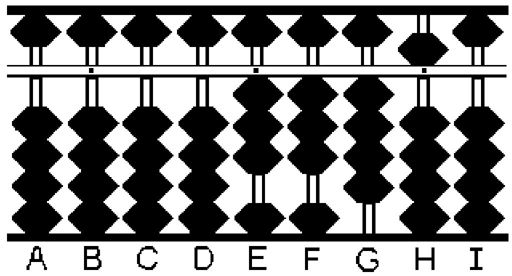
Now we want to add **1** to this. To do so would be simple enough if we had a single available earth bead on Rod H. But all of the earth beads already "have value" because they are pushed towards the reckoning bar. But we still have a heaven bead on rod H that does not "have value" because it is not pushed towards the reckoning bar. But it's value would 5 which is too large – so instead we "Add 5" (by moving the single heaven bead towards the reckoning bar) and "Subtract 4" to give us a net increase of just one. Now we have a frame that looks like:

And you can see the result is **15** (14+1).

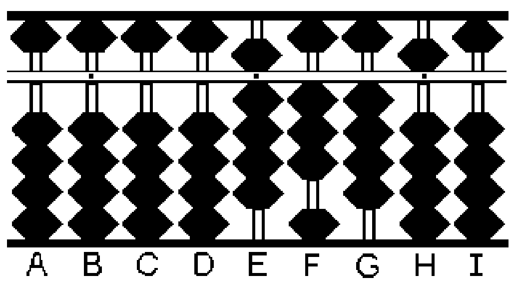


OK – now you are ready for something really meaty. Let's add some serious numbers – no matter how many numbers you deal with, just work with one rod at a time and generally proceed from left to right. Lets try 3345 + 6789 (=10134).

First, **Place 3345** on the frame:

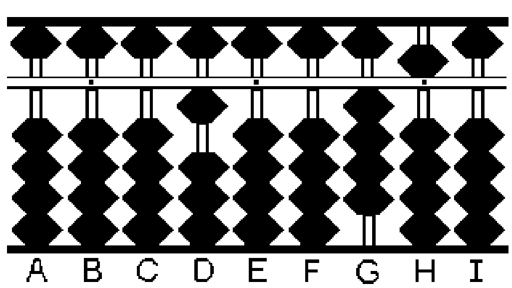


Next (working from left to right) we want to "Add 6" of 6789 on rod E. This is simple enough – simply squeeze the one remaining earth bead and the heaven bead towards the reckoning bar which results in a frame that looks like this:

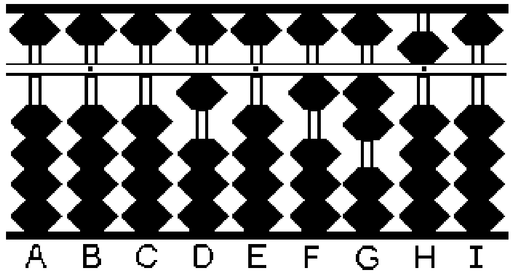


This is the number 9345 but we are not done – there are still three more numbers to go! Next we "Add 7" on rod F. Here we have to carry – a simple glance at the rod will show that there are not 7 worth of beads to move to the reckoning bar. So instead, we "Subtract 3, Add 10" (which is the same as adding 7). We do this by moving down 3 beads on Rod F ("Subtract 3") and then moving up a bead on Rod E (technically "Add 1" but this rod has 10x's the value of rod F so in effect it's like adding 10 to Rod F although the work is done with a single bead on the next column to the left – this is a key concept in the use of the Soroban!).

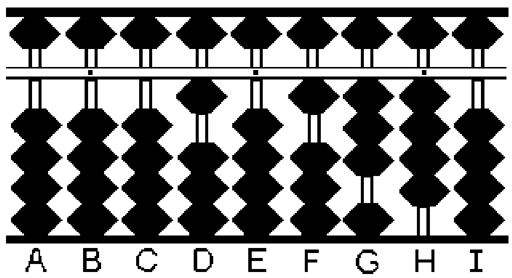
We would be done with this number except that there **are** no free beads on Rod E – so again we must carry to the next rod to the left and "Subtract 9, Add 10" which gives us the same effect. This leaves us with a single bead moved up on Rod D ("Add 1") and nothing on Rod E ("Subtract 9" which is the total of all the beads previously at the reckoning bar on this rod). This results in the following frame:



This is the number 10045. But again, we are not done. Just two more numbers to go. As you can see, the act of carry in addition to the next rod to the left is commonplace with addition – and with practice will become very easy – it can be done as quickly as someone can read off sets of numbers to you. I've found that with just a few weeks of practice, this process becomes very natural. Next we "Add 8" on rod G. Again we don't have enough beads to add 8 so we have to carry. In this case we "Subtract 2" and "Add 10". Therefore we move down two earth beads from rod G and add a single bead to Rod F (which is effectively adding 10 to Rod G because it is one rod to the left). The resulting frame looks like:



This is the number 10125. We are almost done! Just a final digit to add – the "9". So we "Add 9" to the unit rod H. Again, we don't have enough beads to do this with rod H so we must "Subtract 1" and "Add 10". To subtract 1, where we already have a value of 5, we must move away the heaven bead and move the four earth beads to the reckoning bar (essentially "Subtract 5", "Add 4" on this rod). Only after we deal with the unit rod H should we "Add 10" which is accomplished by going to Rod G and adding a single earth bead. When performing addition, always deal with the current rod first – and then if there is a carry move up a single bead on the rod to the left. These are very essential concepts when adding! After performing this step, we have the following resulting frame:

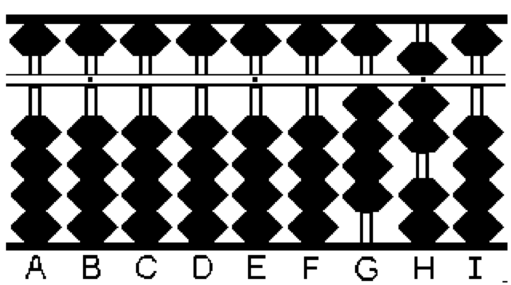


It's easy enough to read this frame and see that the resulting number is **10134** – which is the answer to our original problem (3345 + 6789)! A problem like this may seem strange or even awkward at first – but with a bit of practice, you should be able to work a problem like this in just a few seconds. Even faster and more accurately than you might be able to do it on paper!

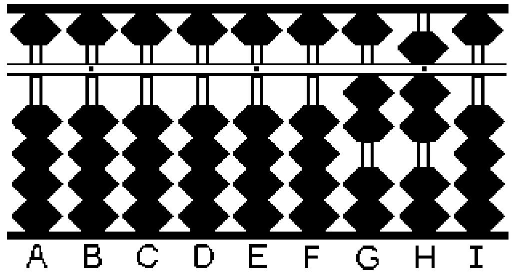
SUBTRACTION

Subtraction on the Abacus is almost as simple as Addition was - it's just the reverse process. Instead of having to deal with a possible carry in the 10's digit (next rod to the left), you now deal with a possible borrow on that same rod. In general, with subtraction you still work the problem and numbers from left to right - just deal with one rod at a time. If there are enough beads that "have value" on the current rod you can just subtract the desired number. If you don't have enough beads with value - you must first subtract 10 (by losing one beads worth of value on the rod to the left of the one you are working on) and then add on your current rod to make up the difference. We will give some concrete examples to show how easy this is.

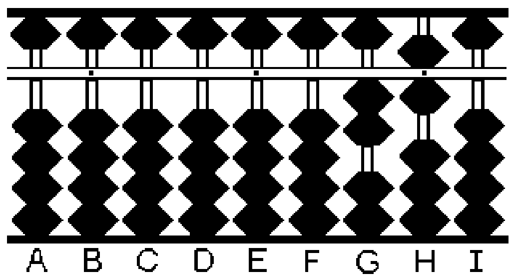
Let us take the number 47 (**Place 47**):



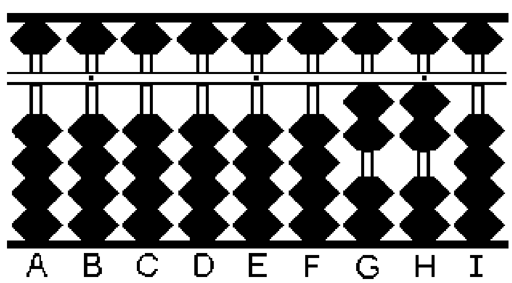
And let's subtract off 21. First we start on Rod G and "Subtract 2" which is easily accomplished by moving 2 earth beads away from the reckoning bar. We now have the value 27 (not the final answer yet):



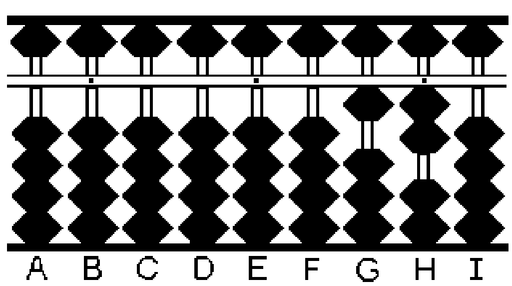
Now we move on to the units rod H and "Subtract 1" by moving a single earth bead away from the reckoning bar. This yields us our final answer of 26:



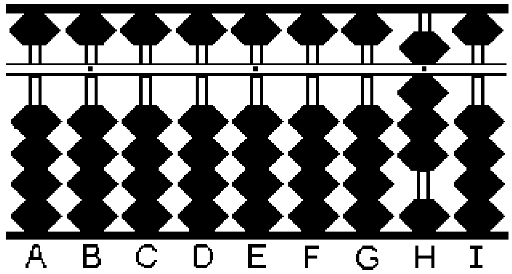
Now let's subtract 4 from this. We can go right to the units rod and "Subtract 4" except that there are not enough single earth beads (worth 1 each) to subtract 4 so instead we must "Subtract 5, Add 1" to give us the same effect. Here we move a single heaven bead (worth 5) away from the reckoning bar and move a single earth bead (worth 1) up to the reckoning bar to accomplish this. We now have our final answer of 22 (which is the result of 26-4):



Now we show how to borrow. Let's take the 22 on the frame and subtract off 14. First we start with rod G and "Subtract 1" which is easy. Our result (not yet the final answer) is:



Now we must "Subtract 4" from our unit rod H. But we don't have 4 worth of beads to subtract on this rod - so instead we must "Subtract 10, Add 6" to produce the same result. To accomplish this, we subtract a single earth bead from rod G (effectively "Subtract 10" since rod G has 10 times the value with respect to rod H which we are working with) and then we "Add 6" to rod H (here we need to move both a Heaven bead and an Earth bead towards the reckoning to accomplish the "Add 6"). This results in a frame with **8** which is our answer!

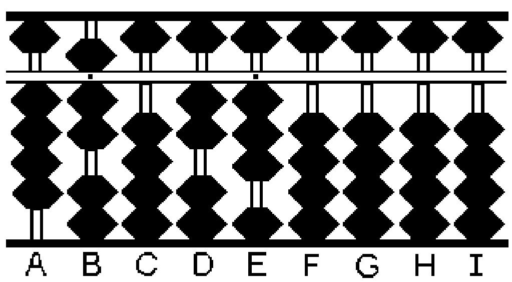


MULTIPLICATION

Multiplication is nothing more than a series of additions. However, it is not very convenient to do 23 separate adds on the number 47 just to give us the answer to 23x47 ! Therefore, there are specific techniques for performing multiplication on the abacus frame. I've learned two different methods and know there are probably others as well – but I will teach only the method that was approved by the Japan Abacus Committee. I have found that this method is least prone to errors and is very simple once you learn the basic technique. You will need to know your multiplication table up to 9x9=81 and you should be all set.

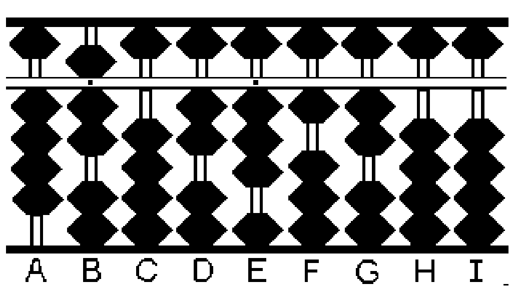
Let's say you are given a problem like 23x47. The number 23 is called the multiplicand and the number 47 is called the multiplier. In general you place the multiplicand (here 23) near the center of the frame – and keep the last digit on a unit rod (marked with a dot) to help keep your place (especially important in multiplication). Then enter the multiplier (here 47) to the left of this number – skipping two clear rods going to the left (do not worry if this number falls properly on a unit dot – it's not necessary). Now you have both numbers on the frame. In the method I will describe you will be producing the answer just to the immediate right of the multiplicand (the number in the middle of your abacus frame). When we are done – the multiplicand (near the center of the frame) will be gone and the answer will remain (along with the multiplier still to the left - though that can be cleared at the end if the Soroban user wishes).

Lets place the 23 and 47 on the frame as we described above (except that I am only going to skip one blank column rod to conserve space in the diagrams! Some prefer skipping just one column and you might have to do it if you have a small abacus – but generally it is better to skip two full columns):

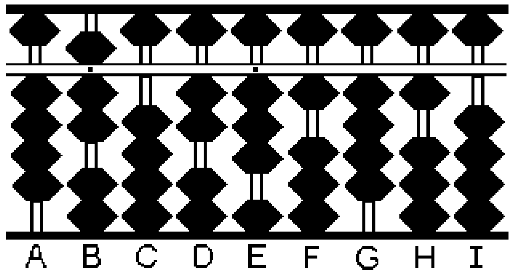


Now we are ready to multiply. It's similar to how you would work it on paper – except that the order of multiplication is a bit different (and should be followed exactly as I outline here).

First you work mostly with the multiplicand (in this case '23'). You take the right-most digit of the multiplicand (in this case a '3') and you multiply it by the left-most digit of the multiplier (in this case a '4'). The result is 12 – and so you add on the frame **two rods to the right** of the multiplicand (in this case FG). Since rods FG have no value, it's just a simple matter of "Add 1" to F and "Add 2" to G to produce the following frame:

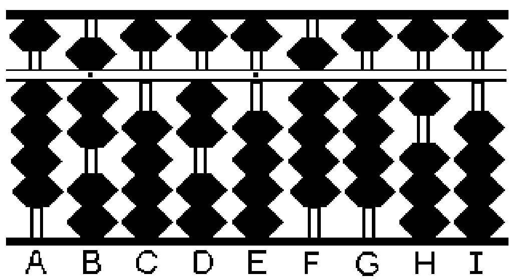


Now we multiply the same multiplicand digit '3' against the next multiplier digit '7' and put that result on GH (since we left off at G after the last multiplication result - we keep moving right so long as there are multiplier digits we are still dealing with). 3x7 is 21 and so we "Add 21" to rods GH which means we "Add 2" to G and "Add 1" to H. Simple enough and here is the resulting frame:

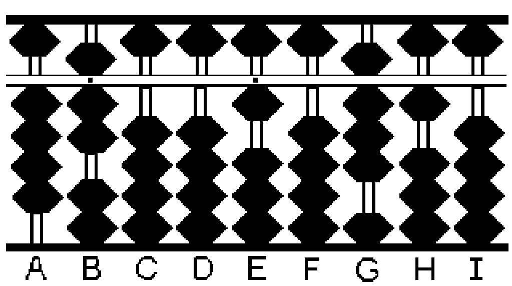


Now we are done with the first digit of the multiplicand and so we must clear it from the frame – we simple clear the '3' off rod E. We will use this rod if there are more numbers in the left in the multiplicand (and in this case there is another number left - we have only done the first digit '3' of the '23' value).

Now we move on to the next digit of the multiplicand – in this case the '2'. We multiply this times the left-most digit of the multiplier (just as we did above for the first digit of the multiplier). And again we place the result (2x4 = 08) on the two rods to the right of multiplicand digit we are working with – rods EF. Important note – the result of 2x4 is a single digit 8 – but enter this as 08 so that you always use up 2 rods so you are technically doing: "Add 0" on E followed by "Add 8" on F. The resulting frame looks like:



We are almost done – we just need to multiply the '2' times the remaining multiplier digit of '7' and add the result of 14 to rods FG (remember we pick up where we left off so long as we haven't moved on to a new multiplicand digit). In this case we "Add 1" to F which requires a carry to E (which you should already know how to do from the addition examples). And then you "Add 4" to G. You clear off the multiplicand digit '2' which has now been processed and see that there are no more multiplicand digits left to process (if there were, you just repeat the above processing). The resulting frame (after clearing away the '2' and you can also clear away the original '47' on the left hand side if you like – but we don't bother do it here) yields the resulting frame:



And so you can read the result of 23x47=**1081** although we still left the 47 on the left side of the frame (you can clear it – I tend to just leave it there). No matter how many digits you multiply, just apply the technique above and remember to work on the correct rod and it will go smoothly.

DIVISION (contributed by Totton Heffelfinger)

For me at least, division was a little intimidating. As it turns out Dave was right when he said, "Trust me. It's not that hard!" As with multiplication you will need to know your multiplication tables up to 9 x 9 = 81. It might be helpful to think of division as being nothing more than a series of subtractions

The techniques I use below are pretty much as they are described in "The Japanese Abacus - It's Use and Theory" by Takashi Kojima.

In describing the methods I will use standard terminology. For example in the problem 8 ÷ 2 = 4, 8 is the dividend, 2 is the divisor, and 4 is the quotient.

Earlier in the handbook Dave wrote about the importance of the Unit rod. (Unit rods are those rods marked with a dot.) Unit rods seem particularly important in solving problems of division because the resulting quotient is often not a perfect whole number. In other words it forms a decimal. Because of this you should plan ahead and set up the problem on your abacus so that the unit number in the quotient falls on a unit rod. Also, in order to help you keep track of your calculations it is a good idea to set the unit number of your dividend on to a unit rod. As for the divisor, if it's a whole number it doesn't seem to matter much whether it follows any such rule.

Normally when setting up division problems on the abacus the dividend is set a little to the right of center and the divisor is set to the left. Traditionally the dividend and divisor are separated by three or four unused rods and this is where the quotient will be formed. Having said that, occasionally I find that four unused rods are not enough and I use more. It really depends on the problem.

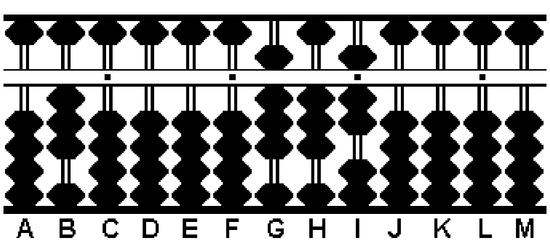
Basically division is done by dividing a single digit into one or possibly two digits at a time. You're required to multiply after each division step and do subtraction to get the remainder. The remainder is then tacked on to the rest of the dividend and the division is continued in this way until its completed. It's not unlike doing it with pencil and paper but the abacus has the advantage of doing much of the grunt work for you.

As you will see I'll set up a few examples with both a whole number and a decimal in the quotient. All is better explained using examples. So let's get started.

Example 1. 837 ÷ 3 = ?

Step 1: Place the dividend 837on the right-hand side of the abacus (in this case on rods G, H & I) and the divisor 3 on the left (in this case rod B). Notice how the "7" in 837 falls on a unit rod.

This yields the resulting frame:

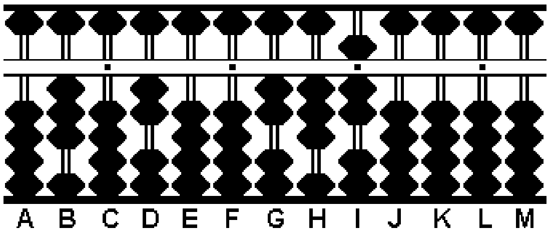



```
Step 1

A B C D E F G H I J K L M
    .     .     .     .
0 3 0 0 0 0 8 3 7 0 0 0 0
```

Step 2: It looks like the quotient is going to have 3 whole numbers in its answer. Begin by placing the first number of the quotient on rod D, that way the unit number in the quotient will fall on unit rod F. In order to divide 837 by 3, start with the 8 in the dividend. It looks like 3 goes into 8 twice with a remainder. Place 2 on rod D. Multiply 2 x 3 to equal 6, then subtract 6 from 8 leaving a remainder of 2.

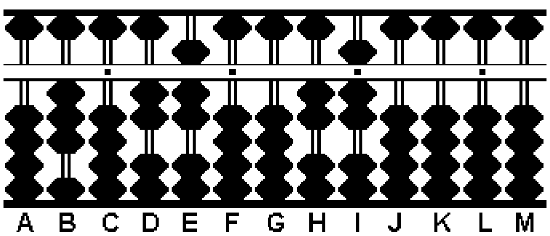
This yields the resulting frame:




```
Step 2
A B C D E F G H I J K L M
    .     .     .     .
0 3 0 0 0 0 8 3 7 0 0 0 0
      2
          - 6
0 3 0 2 0 0 2 3 7 0 0 0 0
```

Step 3: Now there's 237 left with on rods G, H & I. Continue by dividing 23 by 3. It looks like 3 will go into 23 seven times with a remainder. Place 7 on rod E. Multiply 7 x 3 to equal 21, then subtract 21 from 23 leaving a remainder of 2.

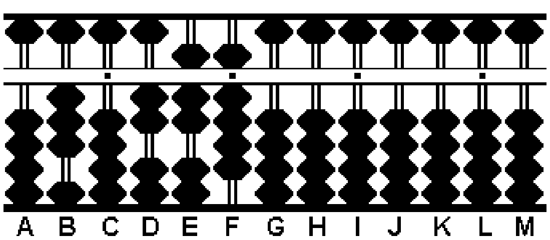
This yields the resulting frame:




```
Step 3
A B C D E F G H I J K L M
    .     .     .     .
0 3 0 2 0 0 2 3 7 0 0 0 0
        7
          - 2 1
0 3 0 2 7 0 0 2 7 0 0 0 0
```

Step 4 and Result: Now there's 27 left on rods H & I. Continue by dividing 3 into 27. 3 goes into 27 nine times perfectly. Place 9 on rod F. Multiply 9 x 3 which equals 27, then subtract 27 from 27 leaving 0. Notice how the "9" in 279 falls on unit rod F.

We're done. **279** is the correct answer

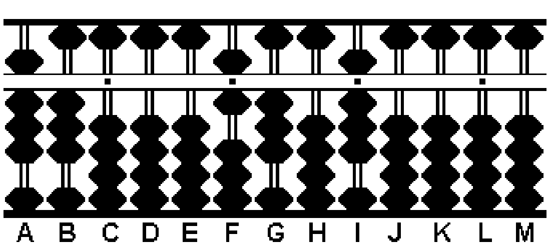



```
Step 4
A B C D E F G H I J K L M
    .     .     .     .
0 3 0 2 7 0 0 2 7 0 0 0 0
          9
            - 2 7
0 3 0 2 7 9 0 0 0 0 0 0 0
```

Example 2. 6308 ÷ 83 = ?

Step 1: Place the dividend 6308 on the right-hand side of the abacus (in this case on rods F, G, H & I) and the divisor 83 on the left (in this case rods A & B). Notice how the "8" in 6308 is placed on a unit rod.

This yields the resulting frame:




```
Step 1

A B C D E F G H I J K L M
    .     .     .     .
8 3 0 0 0 6 3 0 8 0 0 0 0
```

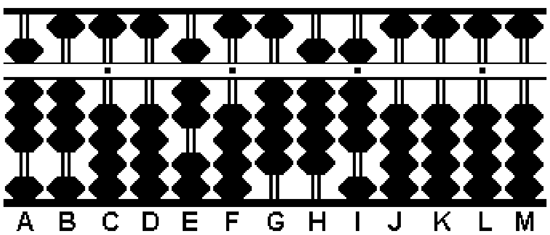
Step 2: In looking at the problem it's evident that 83 will not go into 6 nor will it go into 63. But it will go into 630. It looks like the quotient will have two whole numbers and a possible decimal. Therefore begin to form the quotient on rod E so that the unit number in the quotient will fall on unit rod F.

2a: In order to divide 6308 by 83, start with the 8 in the divisor and the 63 in the dividend. It looks like 8 goes into 63 seven times with a remainder. Place 7 on rod E. Multiply 7 x 8 which equals 56, and subtract 56 from 63 leaving a remainder of 7.

2b: Now there's 708 left on rods G, H & I. Because we've multiplied the "8" in 83 by 7, we must multiply the "3" in 83 by 7. 7 x 3 equals 21. Subtract 21 from 70 leaving a remainder of 49.

```
Step 2a
A B C D E F G H I J K L M
-------------------------
    .     .     .     .
-------------------------
8 3 0 0 6 3 0 8 0 0 0 0 0
        7
        - 5 6
-------------------------
8 3 0 0 7 0 7 0 8 0 0 0 0
```

The resulting frame after steps 2a & 2b:



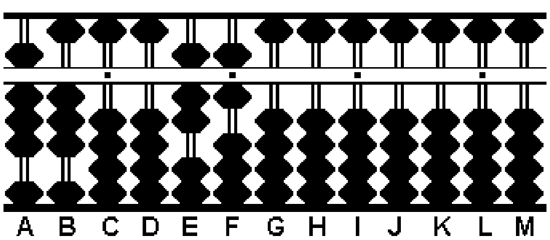

```
Step 2b
A B C D E F G H I J K L M
-------------------------
    .     .     .     .
-------------------------
8 3 0 0 0 0 7 0 8 0 0 0 0
        7
          - 2 1
-------------------------
8 3 0 0 7 0 4 9 8 0 0 0 0
```

Step 3a: Now there's 498 left on rods G, H & I. Continue by dividing the 8 in the divisor into the 49 in the dividend. It looks like 8 goes into 49 six times with a remainder. Place 6 on rod F. Multiply 6 x 8 which equals 48. Subtract 48 from 49 leaving a remainder of 1.

```
Step 3a
A B C D E F G H I J K L M
-------------------------
    .     .     .     .
-------------------------
8 3 0 0 7 0 4 9 8 0 0 0 0
          6
          - 4 8
-------------------------
8 3 0 0 7 6 0 1 8 0 0 0 0
```

3b: Now there's 18 left on rods H & I and we must multiply 6 by the 3 in the divisor. 6 x 3 equals 18. Perfect. Subtract 18 from 18 leaving 0. Notice how the "6" in 76 falls on unit rod F.

We're done. **76** is the correct answer.

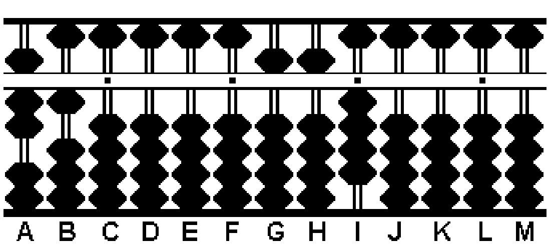



```
Step 3b
A B C D E F G H I J K L M
-------------------------
    .     .     .     .
-------------------------
8 3 0 0 7 0 0 1 8 0 0 0 0
          6
            - 1 8
-------------------------
8 3 0 0 7 6 0 0 0 0 0 0 0
```

Example 3: 554 ÷ 71 = ?

Step 1: Place the dividend 554 on the right-hand side of the abacus (in this case on rods G, H & I) and the divisor 71 on the left (in this case rods A & B).

This yields the resulting frame:




```
Step 1

A B C D E F G H I J K L M
-------------------------
    .     .     .     .
-------------------------
7 1 0 0 0 0 5 5 4 0 0 0 0
```

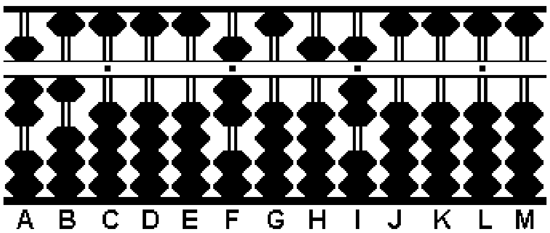
Step 2: In looking at the problem it's evident that 71 will not go into 5 nor will it go into 55. But it will go into 554. It looks like the quotient will have one whole number and a decimal. Therefore begin to form the quotient on rod F. Anything after that will be a decimal.

2a: In order to divide 554 by 71, start with the 7 in the divisor and the 55 in the dividend. It looks like 7 goes into 55 seven times with a remainder. Place 7 on rod F. Multiply 7 x 7 which equals 49, and subtract 49 from 55 leaving a remainder of 6.

```
Step 2a
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 0 5 5 4 0 0 0 0
          7
          - 4 9
7 1 0 0 0 7 0 6 4 0 0 0 0
```

2b: Now there's 64 left on rods H & I and we must multiply 7 times the 1 in the divisor. 7 x 1 equals 7. Subtract 7 from 64 leaving 57.

The resulting frame after steps 2a & 2b:




```
Step 2b
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 7 0 0 0 6 4 0 0 0 0
          7
              - 7
7 1 0 0 0 7 0 5 7 0 0 0 0
```

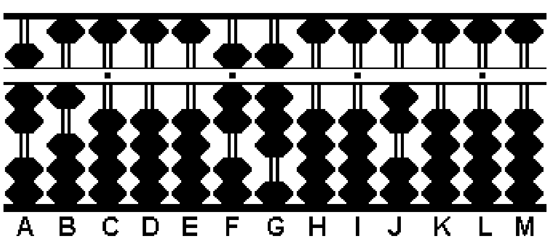
Step 3: Now we're in decimal territory. If we want to continue we're going to have to take a zero from rod J. Let's continue.

3a: Now that we've decided to use a decimal we've got 570 on rods H, I & J. Continue by dividing the 7 in the divisor into the 57 in the dividend .It looks like 7 goes into 57 eight times with a remainder. Place 8 on rod G. Multiply 8 x 7 which equals 56, and subtract 56 from 57 leaving a remainder of 1.

```
Step 3a
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 0 5 7 0 0 0 0
            8
            - 5 6
7 1 0 0 0 7 8 0 1 0 0 0 0
```

3b: We've got 10 left on rods I & J and we must multiply 8 by the 1 in the divisor. 8 x 1 equals 8. Subtract 8 from 10 leaving 2. (Notice how the 8 in the quotient has fallen on the first decimal rod G.)

The resulting frame after steps 3a & 3b:




```
Step 3b
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 0 0 1 0 0 0 0
            8
                - 8
7 1 0 0 0 7 8 0 0 2 0 0 0
```

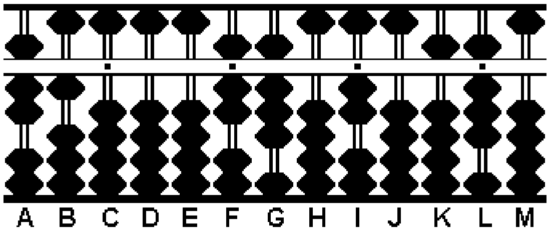
Step 4: Okay. Now there's 2 left on rod J. If we're going to continue we're going to have to take another zero, this time from rod K. That gives us a total of 20. 20 cannot be divided by 71. We must take <u>another</u> zero from rod L. (Note the quotient will have zero on rod H.)

4a: Now there's 200 on rods J, K & L. 200 is divisible by 71. Continue by dividing the 7 in the divisor into the 20 in the dividend. It looks like 7 will go into 20 two times with a remainder. Place 2 on rod I. Multiply 2 x 7 which equals 14. Subtract 14 from 20. That leaves 6.

```
Step 4a
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 8 0 0 2 0 0 0
                2
                - 1 4
7 1 0 0 0 7 8 0 2 0 6 0 0
```

4b: We've got 60 on rods K & L. Now we have to multiply 2 by the 1 in the divisor. 2 x 1 equals 2. Subtract 2 from 60 leaving 58.

The resulting frame after steps 4a & 4b:




```
Step 4b
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 8 0 0 0 6 0 0
                2
                      - 2
7 1 0 0 0 7 8 0 2 0 5 8 0
```

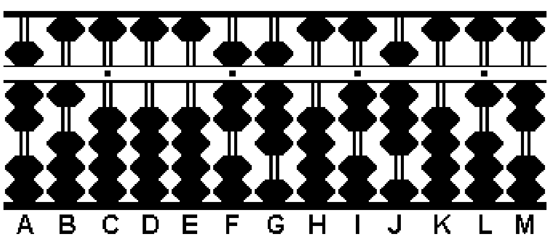
Step 5: Now we've got 58 left on rods K & L. In order to continue we're going to have to take another zero, this time from rod M. (Because we're running out of rods, this will be the final step.)

5a: Now there's 580 on rods K, L & M. Continue by dividing the 7 in the divisor into 58 in the dividend. It looks like 7 will go into 58 eight times with a remainder. Place 8 on rod J. Multiply 8 x 7 which equals 56. Subtract 56 from 58. That leaves 2.

```
Step 5a
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 8 0 2 0 5 8 0
                  8
                  - 5 6
7 1 0 0 0 7 8 0 2 8 0 2 0
```

5b: We've got 20 on rods L & M. Now we have to multiply 8 times the 1 in the divisor. 8 x 1 equals 8. Then subtract 8 from 20 which equals 12. Even though this division problem could go further, this will be as far as we can go because we've run out of room. As with any calculator, the abacus has a limit to how many numbers it can accommodate.

We've arrived at 7.8028 as shown on rods F through J. (The remainder 12 on rods L & M can be ignored.) Rounding off to three decimal points, the answer is **7.803**.




```
Step 5b
A B C D E F G H I J K L M
    .     .     .     .
7 1 0 0 0 7 8 0 2 8 0 2 0
                  8
                        - 8
7 1 0 0 0 7 8 0 2 8 0 1 2
```

That's basically all there is to it. No matter how many digits you find in your divisor or in your dividend, just continue dividing using the above method. For me I find the best way to learn is to use my electronic calculator. (Blasphemous, I know.) What I do is I punch in 3 or 4 random numbers and divide them by 2 or 3 other random numbers. Then without looking at the answer, do the work on my abacus. Then I use the calculator to check my work. My abacus is always way more fun.

ADVANCED OPERATIONS

- *Extracting Square Roots*
- *Use of Decimals for Multiplication and Division*
- *Negative Numbers*

ADDITIONAL CONSIDERATIONS

Daily Practice

The use of the Soroban/Abacus is best learned through practice. Find sets of numbers you can add and subtract - perhaps numbers in a math textbook. Or add successive numbers in the phone book. Or use a computer program to generate lots of random numbers to add. Whichever way you do it - a bit of daily practice will soon make you very comfortable in the use of the abacus. One proven technique for learning addition and subtraction is to take the number "**123456789**" and add this **nine** (9) times. You will get a very neat looking resulting answer (if you did it right!) of "**1111111101**". If you then subtract from this resulting answer "123456789" nine times you will get zero again. Adding and subtracting this special number nine times will teach you nearly every basic manipulation you will ever encounter for addition and subtraction and will improve your speed and accuracy. When I first started it took me about 4 or 5 minutes to add "123456789" nine times and I was only right about half the time. Now I can do it in about 2 minutes and am right about 80% of the time. I continue to practice this and get better and I recommend it as a good "warm-up" exercise when you are using the abacus.

Japanese Soroban Grades/Rankings

The Japanese actually take the use of the Soroban fairly seriously. I'm sure it's gone into a bit of a decline in recent years, but I've read that they teach their children the use of Soroban in the early elementary years and some schools here in the US have given classes on the Soroban and found that they generally improve the math skills of children who learn it. The Japanese also have a ranking system - starting at level 6 (or grade 6) which is somewhat skilled and moving down to level 5 (more skilled) all the way down to level 1 (Soroban mastery). They say that a gifted child who studies the Soroban from an early age can attain a level 1 ranking - while someone who takes up the study of this instrument after they are grown will only ever get to the 3rd or maybe 2nd level. At a level 6 – you must be able to accurately add and subtract sets of 15 different numbers of size 3 or 4 digits. Also, multiplication and division of 2 and 3 digits numbers must be demonstrated. All of this requires a high level of accuracy and each set of numbers is timed. By the time you reach level 3 the numbers have become very large and varied – often mixing 2 and 3 digit numbers with 6 and 7 digit numbers in addition and subtraction. Multiplication and division are now using very large numbers – often 4 or 5 digits and they must also deal with decimal places! Time constraints are much more strict as well! I will try to provide some level 6 practice exams in this handbook in the future for those of you who want to gauge your progress.

RESOURCES

A Where can I buy a Soroban/Abacus?

A Japanese Soroban and/or a Chinese Abacus are not easy to come by locally. There is only one company that I know of that still manufactures new Sorobans – and that would be the Tomoe Soroban Company in Japan (www.soroban.com) . They will sell you a beautiful Soroban – however you need to fax in your order which can be a bit of a pain. Hopefully soon they will have secure web ordering!

News! I just found another website that lists Japanese Sorobans for sale - http://www.citivu.com/usa/sigmaed/ seems to have several of the same makes and models as the Tomoe Soroban Company – and they are based here in the US (California).

As for a Chinese abacus, if you live in a big city which has a Chinatown, you might be able to find a store that sells one of these instruments. Otherwise there are some places online you can check (www.asianideas.com and www.tigergifts.com/abacus.htm and www.onlineoriental.com)

But possibly the best way to pick up a quality Abacus or Soroban is to check the online auction sites such as Ebay (www.ebay.com). I don't want to go into detail on how these auction sites work – and I'll leave it up to you to determine if this route is acceptable. But I can tell you that every week someone seems to be selling an Abacus or Soroban – and often they go very cheaply. I picked up my first 4 Sorobans (3 wooden, 1 plastic all different sizes) for about US$10 each (the plastic which is one of my favorites because of it's size and durability only cost me US$2). A good 13 or 17 column Soroban should be easy to get in the US$10 range – and a stunning vintage 21 or 23 column wooden Soroban should be less than US$20. When you search, be sure to search for both Abacus and Soroban – sometimes the seller does not know which he/she has – and often good Japanese Sorobans are listed under the title of "abacus".

Recommendations for purchasing a Soroban/Abacus.

For a traditional Chinese abacus, I recommend a basic 13 rod frame which is about 12x7 (stay away from the little tiny palm sized ones 2x3 inches). For a Soroban, I recommend a wood frame instrument that is traditional size. For the Japanese Soroban the 13, 17 and 21 are popular sizes (number of rods). The more rods, the larger the numbers it can handle (this is mostly important for multiplication and division – for addition and subtraction smaller instruments would be very adequate). For average use, a 13 column instrument is fine – although a 17 or 21 will allow you do perform nearly any sized calculation. A typical 13 column Soroban is about 7 inches long and about 2.5 inches tall. A big 27 column instrument is about 15 inches long and 2.5 inches tall.

Recommendations for Books about Soroban/Abacus.

There are three books that I have read on the subject (books of this sort are not easy to come by!). The first is still in print – it's called "The Abacus" by Jesse Dilson and comes with a mini wooden Chinese abacus so you can actually use it while learning! The problem is that the book is not the best – some of the techniques are inefficient and the method of multiplication is not as good as the Japanese method. Also, the book is devoted to the Chinese abacus – and has virtually no information on the Japanese Soroban (of course you can easily work one from learning the other). But it is still a good source of information and worth a look.

The second was the The Japanese Abacus Explained by Yoshino, Y. The Yoshino book is fairly good - it covers all the basics of bead manipulation with diagrams (all the older Japanese style with 1 bead on top, 5 on the bottom) and has lots of exercises for addition, subtraction, multiplication and division. Some notes on dictating numbers to someone to add is also given. But overall the book is not as good nor as well written as the Kojima book (see below).

The best book on the use of the Abacus is "The Japanese Abacus – It's Use and Theory" by Takashi Kojima. An all English book published by Charles E. Tuttle company. The book is hard to come by – but the publisher still had some copies in stock last time I checked their website (do a search for Charles E. Tuttle) and I picked up my copy used for about US$10. It's just over 100 pages and absolutely packed with good advice and all practical information on mastering the Abacus (technically the book uses illustrations from a Soroban just the same as my handbook). This book is most excellent – and easy read and very informative. I'd say it's probably the definitive work on the subject!

Recommended Websites.

Tomoe Soroban Company: http://www.soroban.com

The Art of Calculating with the Beads: http://www.ee.ryerson.ca:8080/~elf/abacus/

Soroban from The League of Japan Abacus Associations http://www.syuzan.net/english/index.html

The Abacus: A History: http://fenris.net/~lizyoung/abacus.html

A "Java" Abacus for your use: http://www.mandarintools.com/abacus.html

SOROBAN DISCUSSION GROUP!

I have started a Soroban / Abacus discussion group which is a free email group for the purpose of sparking some discussion on the use and theory of these fascinating calculating tools. You can see the Soroban / Abacus discussion homepage at:

http://groups.yahoo.com/group/SorobanAbacus

Or email me at daveber@gis.net for more details -- I can get you signed up if you are interested – or you can sign up yourself at:

Subscribe to SorobanAbacus

enter email address YAHOO! Groups Join Now!

Powered by groups.yahoo.com